Straight lines and their graphs

All of the material in this chapter is "old news". It was covered in Math102.

*I have included here the material you should be using to graph straight lines. Go through these lecture notes to be certain you know how I expect you to draw the graph of straight lines. We, **all**, will be using the methods outlined in these lecture notes.*

The graphing form for a straight line is $y = mx + b$ where m is the slope and b is the y-intercept. This is also known as the "slope-intercept" form.

We think of the slope *always* as a fraction.

$y = \frac{2}{3}x + 4$ has slope $\frac{2}{3}$. $y = 3x - \frac{1}{2}$ has slope $\frac{3}{1}$.

$y = -2x + 1$ has slope $\frac{-2}{1}$. We always associate the negative with the numerator.

When we want to graph these functions, we

1. Locate the "b" number **on the *y*-axis**.
2. From that point we either count up or down.
   We count up if the numerator is positive.
   We count down if the numerator is negative.
3. Then **from that point** we always count to the right the bottom number.

Points are described as (x_1, y_1) and (x_2, y_2) where the subscript tells from which point the numbers came.

If point 1 is $(5,2)$ and point #2 is $(-15,6)$, then $x_1 = 5$ and $y_2 = 6$.

Two more equations that we will use:

To determine the slope of a straight line: $m = \frac{y_2 - y_1}{x_2 - x_1}$

For our example we have:
$$m = \frac{6-2}{-15-5}$$
$$m = \frac{4}{-20}$$
$$m = \frac{-1}{5}$$

If the slope is negative, we associate the negative with the numerator in our final statement.

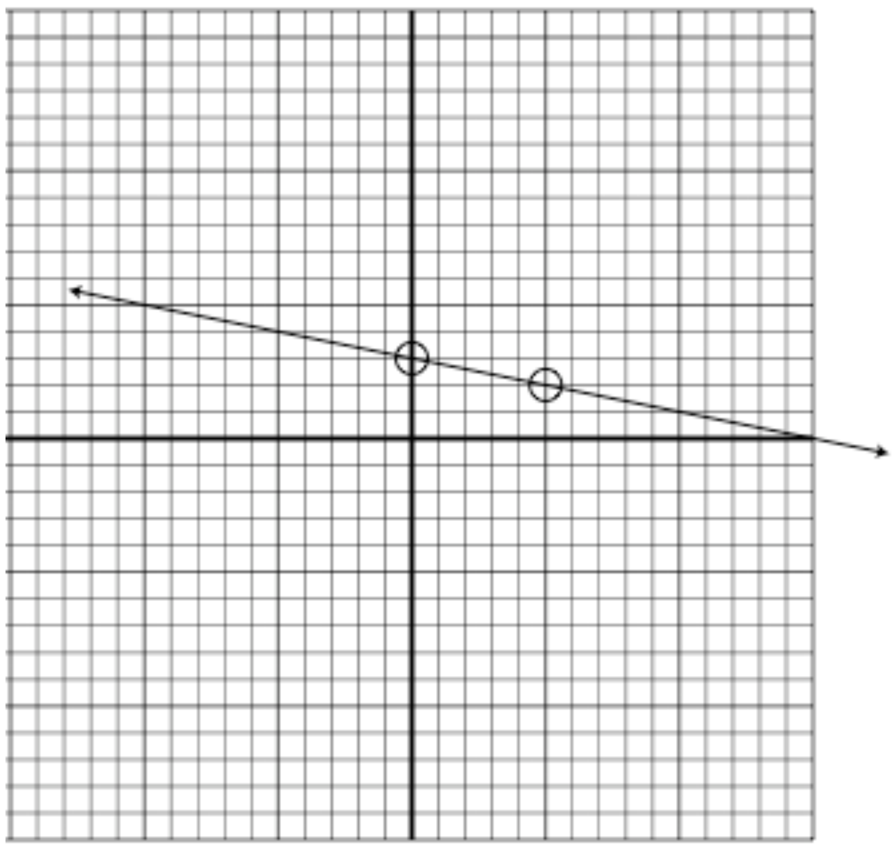
To write the equation of a line given two points, we use: $y - y_1 = m(x - x_1)$.

We will substitute the value of m from above as well as the values for (x_1, y_1).

$$y-2=\frac{-1}{5}(x-5)$$

Thus: $$y-2=\frac{-1}{5}x+1$$

$$y=\frac{-1}{5}x+3$$



Notice the plus 3 is marked. Then I counted down one and five to the right for the second point. Then I drew a line through both points so it would cross both axes.

To summarize: We mark **two** points: the y-intercept and then we counted from that point up(or down) and then counted to the right and marked the second point.

NOTE: This is how we graph straight lines!

It is obvious that $y = 0x + 2$ and $y = \frac{0}{1}$x+2 and $y = 2$ are the same.

You would plot the y-intercept (0, 2). Using the method described above, you would count up zero (that is, not at all) and from that spot you would count right 1 and make your second dot. When you draw the line between the two points, you have a horizontal line. The significance of this is that a horizontal line is a function and has zero slope. I describe a line that contains *only one letter* as a line that "crosses that axis at that point". Our equation crosses only the y-axis and it crosses it at 2.

We can extend the usefulness of this description by considering $x = -3$, for example. This *also* crosses only one axis. It crosses the x-axis and it crosses it at – 3. This graph is of a relation instead of a function since for a single x value we have many y values. It is a vertical line. We say a vertical line "**has no slope**". Notice that is different than saying the line has zero slope!

Consider these two equations: $y = \frac{2}{3}x + 3$ and $y = \frac{2}{3}x - 1$.

If you draw their graphs on the same axis, you will start at different points (+3 for the first one and – 1 for the second equation) but from there you will do exactly the same thing. You will count up 2 and from there you will count 3 to the right. When you draw the lines you will have parallel lines. This shouldn't surprise you since both lines had the same slope.

Consider these two equations: $y = \frac{2}{3}x - 3$ and $y = \frac{-3}{2}x + 5$.

Graph each equation on the same axis. They appear and are, in fact, perpendicular. When the product of the slope of two lines is negative one we say the lines are perpendicular.

Many times this is described as: "A slope of a perpendicular line is the **negative reciprocal** of the slope of the first line."

$\frac{2}{3}$ and $\frac{-3}{2}$ are negative reciprocals.

When we have two equations on the same axis we have one of three situations.

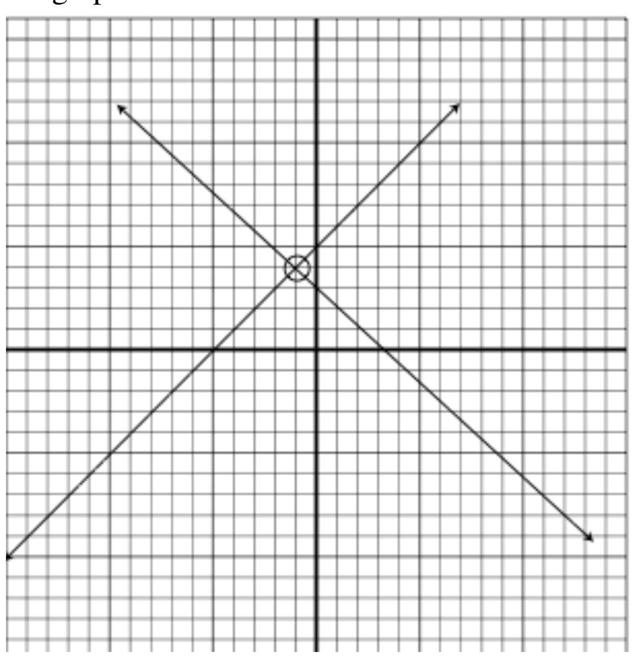
1. The lines intersect in exactly one spot.
2. The lines are parallel and do not intersect at all.
3. Each line is a multiple of the other line and has an identical graph – (i.e. one graph is superimposed on the other.)

The text makes quite a production about “dependent” vs. “independent” and about “consistent” and “inconsistent”. The primary thing they accomplish with their treatment of these is to confuse the student. Basically, those are nice terms but they really have no value for us at this time. You can safely ignore the text’s reference to these terms or questions involving those terms.

The graph of a line is a representation of all those points that satisfy a certain condition. Let us assume that two numbers could be *x* and *y*. Furthermore, let us assume that *y* is larger than *x.*

For example, we could be interested in the numbers that satisfy the condition that the difference of the two numbers is always 5. We would write this as $y - x = 5$. We would re-write this. $y = x + 5$. We could also be interested in all numbers that satisfy the condition that the sum of two numbers is 3. This would be represented as $x + y = 3$. Of course we would re-write this as $y = -x + 3$. These two equations intersect on our graph. Their common intersection (and there is ONLY ONE such point) is $(-1,4)$. Minus one plus four is three and four minus a minus one is five!

We can see this on our graph.



When we have two (or more) equations and we are interested in intersection points, we call that a "system of equations".

We usually write a system of equations as: $\begin{cases} y = x + 5 \\ y = -x + 3 \end{cases}$.

The brace shows all the equations in the "system".

We had no problem seeing the solution. Problems do arise when the intersection is at a fractional place such as $\left(3\frac{17}{23}, -5\frac{2}{15}\right)$. For this reason, "graphing for solution" is the least useful method for solving a system of equations. We will focus on much more effective methods in the next section. For this assignment, the intersections should all be easy to locate.

Remember the instructions for any word problem:

> **Each word problem has a single "Let" statement that defines each of the "participants" in the word problem in terms of a variable.**

Example 1, page 150

Let	w	=	amount of water bought
	S	=	Amount of soda bought

Translating from English to algebra using our definitions above yields:

$$w + S = 76.4$$
$$w = \frac{1}{2}S - 4.3$$